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CHARACTERIZATION OF THE SOLUTIONS OF INTERVAL SYSTEM OF LINEAR EQUATIONS OVER INTERVAL SUPERTROPICAL ALGEBRA

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Abstrak

Penelitian ini bertujuan untuk mediskusikan mengenai solusi dari sistem persamaan interval atas aljabar supertropical. Aljabar supertropical merupakan semiring komutatif dengan ghost. Sistem persamaan linear interval pada aljabar supertropical diberikan oleh persamaan matriks $A\overline{\otimes}\, x \vDash b$ dengan $\boldsymbol{A} = \langle \underline{A}, \overline{A} \rangle$ adalah matriks interval $\boldsymbol{b} = \langle \underline{b}, \overline{b} \rangle$ adalah vector interval, dan $\mathrm{x} = \langle \underline{\mathrm{x}}, \overline{\mathrm{x}} \rangle$ merupakan vektor interval yang merupakan solusi dari system persamaan linear interval. Pada peneletian ini ditunjukkan syarat perlu dan syarat cukup dari solusi sistem persamaan linear interval serta ditunjukkan bahwa terdapat dua tipe solusi dari sistem persamaan linear interval atas aljabar supertropical.

Abstract

In supertropical algebra, the focus our study is to utilize the principle of supertropical algebra in order to solve a system of interval equations. Supertropical algebra is a commutative semiring with ghost. The interval linear equation system in supertropical algebra is given by a matrix equation $\boldsymbol{A}\overline{\otimes}\, \boldsymbol{x} \vDash \boldsymbol{b}$, *In this context, the matrix can be expressed as an interval matrix denoted by* $\mathrm{A} = \langle \underline{\mathrm{A}}, \overline{\mathrm{A}} \rangle$ *,while the vector* $\boldsymbol{b}$ *can be represented as* $\mathrm{b} = \langle \underline{\mathrm{b}}, \overline{\mathrm{b}} \rangle$ *and is an interval vector, and the interval vector used to solve the solution system is* $\mathrm{x} = \langle \underline{\mathrm{x}}, \overline{\mathrm{x}} \rangle$ *. In this study to identify the,necessary and sufficient conditions requirements to obtain the solution of an interval linear equations and the analysis of the interval linear equation system utilizing supertropical algebra the presence of two distinct solution types.*

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INTRODUCTION

Maxplus algebra is a sub class of tropical algebra, tropical algebra is an idempotent semiring and semifield (Yuliati, 2016). Many problems in optimization on networks, discrete event dynamic system are linear in the maxplus algebra, many phenomena are modeled as linear system. The typical representation of network modelling using the maxplus algebra approach involves a matrix equation that describes a system of maxplus linear equations $A\overline{\otimes}\, x = b$ where x represents the input, while the vector b represents the output. Within this algebra, there have been several investigations in the literature regarding different types of solvability conditions for interval linear systems. One of the problems such that solving of system of linear equations over maxplus algebra was published in (Kurniawan et al., 2020). Research by Schutter described the explore several findamental characteristic of maxplus algebra and demonstrate how these conditions can be employed to analyze the behaviour of maxplus linear discrete event systems. Additionally Schutter provide an overview of various control methods for these systems, including model predictive control, and touch upon some expansions of the maxplus algebra and maxplus linear systems (Komenda et al., 2017). Paper by Akian focuses on the examination of the system linear of equations over maxplus algebra (Akian et al., 1990). Paper by Lihua Wang presents the notion of solutions for two sided interval maxplus linear systems, which is a broad encompassing various known solutions to interval linear system (Wang et al., 2018).

Supertropical algebra is new structure of commutative semiring, that is covers the maxplus algebra is endowed with ghost surpassing relation (Izhakian & Rowen, 2010b). Recently, the supertropical algebra have been developed. Research by Izhakian and Rowen (2010), the development of the theory of supertropical algebra includes the introduction of ghost elements and the factoring of any polynomial into linear and quadratic factors. The research of Adi Niv also studied the characteristic polynomials of the tropical similarity matrix and of the polynomial of pseudoinverses (Niv, 2015). Research on Izhakian and Rowen (2011) create a general theory of supertropical matrix algebra about the tropical determinant, adjoint matrix, eigen value and eigen vector of matrix over supertropical algebra(Izhakian & Rowen, 2011a). In Supertropical matrix algebra III, Izhakian focus investigation on the powers of supertropical matrices, with particular emphasis on how the supetropical trace and other coefficients of the supertropical characteristic polynomial can influence and control the rank of a matrix power (Izhakian & Rowen, 2011b).

Research by Dian (2016) described the necessary and sufficient conditions solutions of non homogeneus system of linear equation in supertropical algebra. In this paper, we will to replace elements of system of maxplus linear equation $A\overline{\otimes}\, x \vDash b$ by intervals. A system of linear equations with interval is provided, which is gives rise to a resulting system. This resulting system consist of maxplus linear equations with intervals have been studied in (Myšková, 2005). Other solvability concepts of interval system of maxplus linear equation are have been studied in(Myšková, 2012). On occasion, a system of linear equation may not have a solution. This paper aims to investigate the solutions interval linear equations in supertropical algebra and find a resolution for the interval system of maxplus linear equations. The study will present necessary and sufficient conditions for obtaining the solution of an interval linear equation system.

METHOD

In this basic concepts about matrices in supertropical algebra related to matrices and system of linear equations.

2.1 Maxplus Algebra

Definition 2.1.1. The maxplus algebra is defined as the collection of elements $\mathbb{R} \cup \{\varepsilon\}$ and operations addition and the multiplication that satisfy certain properties, for every $a, b \in \mathbb{R}$ (Jones, 2021)

$a \oplus b = max\{a, b\}$ and $a \otimes b = a + b$ (1)

Where $\varepsilon = -\infty$ and $e = 0$.

Maxplus algebra is an example of tropical algebra and is written as $\mathbb{R}_{max}$. Let $\mathbb{R}_{max}^{mxn}$ as the collection of all elements mxn matrix whose entries belong to $\mathbb{R}_{max}$. The operations $\oplus$ and $\otimes$ on $\mathbb{R}_{max}$ can also include the set in $\mathbb{R}_{max}^{mxn}$, where $\mathbb{R}_{max}^{mxn} = \{A = (A_{ij}) | A_{ij} \in \mathbb{R}_{max}, i = 1,2, \dots, m \text{ } and \text{ } j = 1,2, \dots, n\}$. The order relation $\preccurlyeq$ on $\mathbb{R}_{max}^{mxn}$ that is only defined on certain pairs of elements within a set $A \preccurlyeq B \leftrightarrow A_{ij} \preccurlyeq B_{ij}, 1 \le i \le m, 1 \le j \le n$ for every $A, B \in \mathbb{R}_{max}^{mxn}$.

Definition 2.1.2. For $A, B \in \mathbb{R}_{max}^{mxn}$ and $\alpha \in \mathbb{R}$ we define operation

$(A \oplus B)_{ij} = a_{ij} + b_{ij}$ and $\alpha \otimes B_{ij} = \alpha \otimes b_{ij}$ (2)

Definition 2.1.3. For $A \in R_{max}^{nxp}$ and $B \in \mathbb{R}_{max}^{pxm}$ we define operation

$(A \otimes B)_{ij} = \oplus_{k=1}^{n} a_{ik} \otimes b_{kj}$ (3)

Define $A \in \mathbb{R}_{max}^{n} = \{[x_1, x_2, \dots, x_n]^T\} | x_i \in \mathbb{R}_{max}, i = 1,2, \dots, n\}$. It should be noted that can be seen as $A \in R_{max}^{nx1}$. The elements of $\mathbb{R}_{max}^{n}$ are called vectors over $\mathbb{R}_{max}$.

2.2 Supertropical Algebra

A semiring $(\mathcal{R}, \mathcal{G}_0, v)$ that includes a ghost is defined by the semiring R with $(0_R = \varepsilon = -\infty \text{ } and \text{ } 1_R = e = 0,)$ and an ideal $\mathcal{G}_0$ with a ghost mapping $v : R \to \mathcal{G}_0$. The mapping satisfies the condition that $a \oplus a = v(a), \forall a \in R$ and a^v denotes $v(a)$.

Definition 2.2.1. Supertropical algebra can be described as a semiring with ghost $(\mathcal{R}, \mathcal{G}_0, v)$ that satisfies additional properties for every $a, b \in R$

1. $a \oplus b = \{a, b\} \text{ } if \text{ } a \neq b$
2. $a \oplus b = a^v \text{ } if \text{ } a = b$ (4)

$\mathcal{T} = R \backslash \mathcal{G}_0$ is tangible elements and $\mathcal{G}$ is ghost elements.

The v -order on R is defined as follows

$a \succcurlyeq_v b \leftrightarrow a^v \succcurlyeq b^v$ and $a \succ_v b \leftrightarrow a^v \succ b^v$ (5)

Definition 2.2.2. The ghost surpasses relation on R is defined by the following relation $\vDash$ (Izhakian & Rowen, 2010a)

a $\vDash$ b if $a = b \oplus g$ for some $g \in \mathcal{G}_0$ (6)

When $a \vDash b$ then $a \oplus b \in \mathcal{G}_0$

Definition 2.2.3. The definition of partial order relation $\prec$ on T is as follows (Izhakian, 2019)

1. $-\infty \prec a, \forall a \in \mathcal{T} \backslash -\infty$
2. $a \prec b \text{ } then \text{ } a \prec b^v, a^v \prec b \text{ } and \text{ } a^v \prec b^v$ for every real numbers
3. $a \prec a^v$ for every $a \in R$

Definition 2.2.4. For any semiring R we have the semiring $M_n(R)$ of $n \times n$ matrices for every $n \in N$, with $n \neq 0$ and translated into(Izhakian & Rowen, 2010a)

$$M_n(R) = \left\{ \begin{bmatrix} a_{1,1} & \cdots & a_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \cdots & a_{n,n} \end{bmatrix} \middle| a_{i,j} \in R \right\} \quad (7)$$

Where $a_{i,j}$ is an element of $A \in M_n(R)$ row-i and column-j.

Relation ghost surpasses over matrix is defined by $A \vDash B$ if $A = B \oplus G$ for some $\in M_n(\mathcal{G}_0)$. Identity matrix over supertropical algebra is defined by

$$[I_R]_{i,j} = \begin{cases} e, & for\ i = j \\ \varepsilon, & for\ i \neq j \end{cases} \tag{8}$$

Supertropical determinant of matrices $A \in M_n(R)$ to be permanent, i.e.,

$$|A| = \bigoplus_{\sigma \in S_n} a_1 \sigma_1 \dots a_n \sigma_n \tag{9}$$

Let $A \in M_n(R)$, minor $M_{i,j}$ of an element in a matrix. Cofactor of an element $a_{i,j}$ is defined by $C_{i,j} = (-1)^{i+j} M_{i,j}$ where $M_{i,j}$ is a minor of $a_{i,j}$ we can define adjoint matrix of A as follows

$$adj(A) = (cof(A))^T \text{ where } cof_{i,j} = M_{i,j}. \tag{10}$$

Let $A \in M_n(R)$ is a non singular matrix over supertropical algebra if $|A| \in \mathcal{T}$, and is a singular matrix over supertropical algebra if $|A| \in \mathcal{G}_0$.

Definition 2.2.5. The set that consist of the closed interval in a in $\mathbb{R}_{max}$ is a subset of $\mathbb{R}_{max}$, $\boldsymbol{a} = \langle \underline{a}, \overline{a} \rangle = \{a \in \mathbb{R}_{max} | \underline{a} \preccurlyeq a \preccurlyeq \overline{a}\}$ where $\underline{a}, \overline{a} \in \mathbb{R}_{max}$, are the lower and upper limits of the interval are referred to as the lower and upper bounds a**,** respectively **.** $\boldsymbol{a}$ The interval in $\mathbb{R}_{max}$ is called maxplus interval (Rudhito et al., 2012). Define

$$I(\mathbb{R})_\varepsilon = \{\boldsymbol{a} = \langle \underline{a}, \overline{a} \rangle | \underline{a}, \overline{a} \in R, \varepsilon \preccurlyeq \underline{a} \preccurlyeq \overline{a}\} \cup [\{\varepsilon, \varepsilon\}] \tag{11}$$

In the $I(\mathbb{R})_\varepsilon$ define Operation $\oplus$and $\otimes$ as

$$\boldsymbol{a} \oplus \boldsymbol{b} = [\underline{a} \oplus \underline{b}, \overline{a} \oplus \overline{b}] \text{ and } \boldsymbol{a} \otimes \boldsymbol{b} = [\underline{a} \otimes \underline{b}, \overline{a} \otimes \overline{b}] \text{ for any } \boldsymbol{a}, \boldsymbol{b} \in I(\mathbb{R})_\varepsilon \tag{12}$$

$I(\mathbb{R})_\varepsilon$ is an idempotent commutative semiring $\boldsymbol{\varepsilon} = [\varepsilon, \varepsilon], \boldsymbol{0} = [0, 0]$ Moreover $(I(\mathbb{R})_\varepsilon, \overline{\oplus}, \overline{\otimes})$is called interval maxplus algebra and denoted by $I(\mathbb{R})_{max}$.

$$I(\mathbb{R})_{max}^{m \times n} = \{A | A_{i,j} \in I(\mathbb{R})_{max},\ for\ i = 1,2, \dots, m\ and\ j = 1,2, \dots, n\}$$

The element of $I(\mathbb{R})_{max}^{m \times n}$ called matrices over interval maxplus algebra. The interval matrix in $\mathbb{R}_{max}$ is

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{a}_{11} & \cdots & \boldsymbol{a}_{1n} \\ \vdots & \ddots & \vdots \\ \boldsymbol{a}_{m1} & \cdots & \boldsymbol{a}_{mn} \end{bmatrix} = \begin{bmatrix} \langle \underline{a_{11}}, \overline{a_{11}} \rangle & \cdots & \langle \underline{a_{1n}}, \overline{a_{1n}} \rangle \\ \vdots & \ddots & \vdots \\ \langle \underline{a_{m1}}, \overline{a_{m1}} \rangle & \cdots & \langle \underline{a_{mn}}, \overline{a_{mn}} \rangle \end{bmatrix}$$

Where $A_{i,j} \in \mathbb{R}_{max}$. Let $\underline{A} = (\underline{A}_{i,j}), \overline{A} = (\overline{A}_{i,j})$ where $\underline{A}, \overline{A} \in (\mathbb{R})_{max}^{m \times n}$ then $\boldsymbol{A} = \langle \underline{A}, \overline{A} \rangle = \{A \in (\mathbb{R})_{max}^{m \times n} | \underline{A} \preccurlyeq A \preccurlyeq \overline{A}\}$. $I(\mathbb{R})_{max}^{m \times n}$ denotes the set of all $m \times n$ interval matrix in $\mathbb{R}_{max}$. The operation $\overline{\oplus}$ and $\overline{\otimes}$ in $I(\mathbb{R})_{max}$ can be extended to the matrix operations in $I(\mathbb{R})_{max}^{m \times n}$.

Definition 2.2.6 Let $\boldsymbol{A} = \langle \underline{A}, \overline{A} \rangle$ and $= \langle \underline{b}, \overline{b} \rangle$, the system of linear equations in the maxplus algebra that involves intervals is represented by $A \overline{\otimes} x = b$, a solution to this system is provided in the form of an interval vector $\boldsymbol{x} = \langle \underline{x}, \overline{x} \rangle$.

RESULT AND DISCUSSION

The conditions that are both necessary and sufficient conditions for finding a solution to an interval system of linear equations over supertropical algebra are covered in this section. Interval system of linear equations $A \otimes x = b$ will be weakned with relation ghost surpasses $A \otimes x \vDash b$.

Proposition 3.1. *Let $A \in M_n(R)$, $b \in \mathcal{T}_0^n$, and $x \in R^n$ System of linear equations $\boldsymbol{A} \otimes x \vDash$ has a unique tangible solution if and only if $|A| \in \mathcal{T}$ and $(adj(A) \otimes b) \in \mathcal{T}_0^n$.*

Proposition 3.1 provide the solutions for a non homogeneus system of linear equations over supertropical algebra $A \otimes x \vDash b$. Moreover, we will to replace elements of matrix A and vector b by interval matrix $\boldsymbol{A} = \langle \underline{A}, \overline{A} \rangle$ and interval vector $\boldsymbol{b} = \langle \underline{b}, \overline{b} \rangle$, and interval system of supertropical linear equation is represent by $A \overline{\otimes} x = b$.

Definition 3.1. Interval system of linear equations over supertropical algebra $\boldsymbol{A} \overline{\otimes} x \vDash \boldsymbol{b}$ has a solutions **k** if $\boldsymbol{A} \overline{\otimes} k \vDash \boldsymbol{b}$ where $\boldsymbol{k} = \langle \underline{k}, \overline{k} \rangle$ such that $\underline{A} \otimes \underline{k} \vDash \underline{b}$ and $\overline{\boldsymbol{A}} \otimes \overline{k} \vDash \overline{\boldsymbol{b}}$.

Theorem 3.1. If interval system of linear equations $\boldsymbol{A} \overline{\otimes} x \vDash \boldsymbol{b}$ has a solutions $\boldsymbol{k} = \langle \underline{k}, \overline{k} \rangle$ for $\boldsymbol{A} \in \langle \underline{A}, \overline{A} \rangle$ and $\boldsymbol{k} \in \langle \underline{k}, \overline{k} \rangle$ then $\overline{\otimes} k \in \langle \underline{b}, \overline{b} \rangle$.

Proof. We will proof that $\boldsymbol{A} \overline{\otimes} k \in \langle \underline{b}, \overline{b} \rangle$.

Hence $\underline{A} \preccurlyeq_v A$ then $\underline{A} \oplus A = max\{\underline{A}, A\} = A$ and $A \preccurlyeq_v \overline{A}$ then $A \oplus \overline{A} = max\{A, \overline{A}\} = \overline{A}$

$(\underline{A} \oplus A) \otimes k = \underline{A} \otimes k \oplus A \otimes k$

$A \otimes k = \underline{A} \otimes k \oplus A \otimes k$

If $P = A \otimes k$ and $Q = \underline{A} \otimes k$ then $= Q \oplus P$, so $Q \preccurlyeq_v P$ equivalent with $\underline{A} \otimes k \preccurlyeq_v A \otimes k$

$(A \oplus \overline{A}) \otimes k = A \otimes k \oplus \overline{A} \otimes k$

$\overline{A} \otimes k = A \otimes k \oplus \overline{A} \otimes k$

If $S = \overline{A} \otimes k$ and $P = A \otimes k$ then $= P \oplus S$, so $P \preccurlyeq_v S$ equivalent with $A \otimes k \preccurlyeq_v \overline{A} \otimes k$

Use the partial relation then

$Q \preccurlyeq_v P \preccurlyeq_v S$

$\underline{A} \otimes k \preccurlyeq_v A \otimes k \preccurlyeq_v \overline{A} \otimes k$

Hence $\underline{k} \preccurlyeq_v k$ then $\underline{k} \oplus k = \max\{\underline{k}, k\} = k$ and $k \preccurlyeq_v \overline{k}$ then $k \oplus \overline{k} = max\{k, \overline{k}\} = \overline{k}$

$\underline{A} \otimes (\underline{k} \oplus k) = \underline{A} \otimes \underline{k} \oplus \underline{A} \otimes k$

$\underline{A} \otimes k = \underline{A} \otimes \underline{k} \oplus \underline{A} \otimes k$

If $B = \underline{A} \otimes k$ and $C = \underline{A} \otimes \underline{k}$ then $= C \oplus B$, so $C \preccurlyeq_v B$ equivalent with $\underline{A} \otimes \underline{k} \preccurlyeq_v \underline{A} \otimes k$

$\overline{A} \otimes (\underline{k} \oplus k) = \overline{A} \otimes \underline{k} \oplus \overline{A} \otimes k$

$\overline{A} \otimes k = \overline{A} \otimes \underline{k} \oplus \overline{A} \otimes k$

If $D = \overline{A} \otimes k$ and $E = \overline{A} \otimes \underline{k}$ then $= E \oplus D$, so $E \preccurlyeq_v D$ equivalent with $\overline{A} \otimes \underline{k} \preccurlyeq_v \overline{A} \otimes k$

Use the partial relation then

$$C \preccurlyeq_v B \preccurlyeq_v E \preccurlyeq_v D$$
$$\underline{A} \otimes \underline{k} \preccurlyeq_v \underline{A} \otimes k \preccurlyeq_v \overline{A} \otimes \underline{k} \preccurlyeq_v \overline{A} \otimes k$$
$$\underline{A} \otimes \underline{k} \preccurlyeq_v \underline{A} \otimes k \preccurlyeq_v A \otimes k \preccurlyeq_v \overline{A} \otimes \underline{k} \preccurlyeq_v \overline{A} \otimes k$$

From the Definition that

$\underline{A} \otimes \underline{k} \vDash b$ and $\overline{A} \otimes \overline{k} \vDash \overline{b}$ then

$\underline{b} \preccurlyeq_v A \otimes k \preccurlyeq_v \overline{b}$ so $A \otimes k \in \langle \underline{b}, \overline{b} \rangle$

Theorem 3.2. Let $\underline{A} \in M_n(R), \overline{A} \in M_n(R)$, $\underline{b} \in \mathcal{T}_0^n$, $\overline{b} \in \mathcal{T}_0^n$and $\underline{x} \in R^n$, $\overline{x} \in R^n$ Interval system of linear equations over supertropical algebra $\boldsymbol{A} \overline{\otimes} \boldsymbol{x} \vDash \boldsymbol{b}$ has a unique tangible solution if and only if $|\underline{A}| \in \mathcal{T}, |\overline{A}| \in \mathcal{T}$ and $(adj(\underline{A}) \otimes \underline{b}) \in \mathcal{T}_0^n, (adj(\overline{A}) \otimes \overline{b}) \in \mathcal{T}_0^n$.

Proof. By the definition 3.1 interval system $\boldsymbol{A} \overline{\otimes} \boldsymbol{x} \vDash \boldsymbol{b}$ has a solutions then system $\underline{\boldsymbol{A}} \otimes \underline{x} \vDash \underline{\boldsymbol{b}}$ and $\overline{\boldsymbol{A}} \otimes \overline{x} \vDash \overline{\boldsymbol{b}}$ respectively has a unique tangible solution, when to use proposition 3.1 obtained

$|\underline{A}| \in \mathcal{T}, |\overline{A}| \in \mathcal{T}$ and $\left(adj(\underline{A}) \otimes \underline{b}\right) \in \mathcal{T}_0^n, \left(adj(\overline{A}) \otimes \overline{b}\right) \in \mathcal{T}_0^n$.

From theorem 3.2 given a necessary and sufficient conditions interval system $\boldsymbol{A} \overline{\otimes} \boldsymbol{x} \vDash \boldsymbol{b}$ has an interval unique tangible solution if and only if $|\underline{A}| \in \mathcal{T}, |\overline{A}| \in \mathcal{T}$ and $\left(adj(\underline{A}) \otimes \underline{b}\right) \in \mathcal{T}_0^n, \left(adj(\overline{A}) \otimes \overline{b}\right) \in \mathcal{T}_0^n$. If condition not fulfilled then interval system $\boldsymbol{A} \overline{\otimes} \boldsymbol{x} \vDash \boldsymbol{b}$ has an interval infinite number of solutions.

Corollary 3.1. *Let* $\underline{A} \in M_n(R), \overline{A} \in M_n(R)$, $\underline{b} \in \mathcal{T}_0^n$, $\overline{b} \in \mathcal{T}_0^n$ and $\underline{x} \in R^n, \overline{x} \in R^n$ *Interval system of linear equations over supertropical algebra* $\boldsymbol{A} \overline{\otimes} \boldsymbol{x} \vDash \boldsymbol{b}$ has infinitely many solutions *if and only if* $|\underline{A}| \in \mathcal{T}, |\overline{A}| \in \mathcal{T}$ *and* $\left(adj(\underline{A}) \otimes \underline{b}\right) \notin \mathcal{T}_0^n, \left(adj(\overline{A}) \otimes \overline{b}\right) \notin \mathcal{T}_0^n$ *or* $|\underline{A}| \in \mathcal{G}_0 \neq \varepsilon, |\overline{A}| \in \mathcal{G}_0 \neq \varepsilon$ *and* $\left(adj(\underline{A}) \otimes \underline{b}\right) \notin \mathcal{T}_0^n, \left(adj(\overline{A}) \otimes \overline{b}\right) \notin \mathcal{T}_0^n$ *or* $|\underline{A}| \in \mathcal{G}_0 \neq \varepsilon, |\overline{A}| \in \mathcal{G}_0 \neq \varepsilon$.

CONCLUSION

The conditions for determining criteria the existence and uniqueness of interval solutions for system of interval linear equations over supertropical algebra $\boldsymbol{A} \overline{\otimes} \boldsymbol{x} \vDash \boldsymbol{b}$ are as follows, the interval system $\boldsymbol{A} \overline{\otimes} \boldsymbol{x} \vDash \boldsymbol{b}$ has a unique tangible solution if and only if $|\underline{A}| \in \mathcal{T}, |\overline{A}| \in \mathcal{T}$ and $\left(adj(\underline{A}) \otimes \underline{b}\right) \in \mathcal{T}_0^n, \left(adj(\overline{A}) \otimes \overline{b}\right) \in \mathcal{T}_0^n$ while it has infinitely many solutions if and only if $|\underline{A}| \in \mathcal{T}, |\overline{A}| \in \mathcal{T}$ and $\left(adj(\underline{A}) \otimes \underline{b}\right) \notin \mathcal{T}_0^n, \left(adj(\overline{A}) \otimes \overline{b}\right) \notin \mathcal{T}_0^n$ or $|\underline{A}| \in \mathcal{G}_0 \neq \varepsilon, |\overline{A}| \in \mathcal{G}_0. \neq \varepsilon$ and $\left(adj(\underline{A}) \otimes \underline{b}\right) \notin \mathcal{T}_0^n, \left(adj(\overline{A}) \otimes \overline{b}\right) \notin \mathcal{T}_0^n$ *or* $|\underline{A}| \in \mathcal{G}_0 \neq \varepsilon, |\overline{A}| \in \mathcal{G}_0 \neq \varepsilon$.

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