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ANALYSIS DINAMIC AND BIOECONOMIC OF A PREDATOR-PREY SYSTEM WITH MARINE NATIONAL PARK

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Abstrak

Pada kajian perikanan atau hasil laut pada negara berkembang sering dihadapkan pada persoalan pengeloaan hasil sumber daya laut. Ekspoitasi yang berlebihan karena lemahnya sektor hukum dan pengawasan menjadi faktor yang paling sering digunakan. Riset ini melibatkan pemodelan matematika predator-prey dan memberikan intervensi variabel eksploitasi yaitu pemanenan. Pemanenan dilakukan pada dua jenis spesies yang mendiami dua wilayah zona taman nasional laut yang di lindungi. Salah satu tujuan variabel eksploitasi adalah memberikan keuntungan bagi pelaku pemanenan, seperti nelayan. Wilayah batas pada zona taman nasional laut dan titik equilibrum diberikan pada pembasahan riset. Analisis kestabilan menggunakan Routh-Hurwitz kriteria yang menandakan keberlangsungan kehidupan population. Model predator-prey yang dibentuk menghasilkan tujuh equilibrium non-negatif, akan tetapi hanya satu titik equilibrium yang memenuhi kaidah asumsi penelitian. Simulasi numerik juga diberikan secara trajektories dari awal pembentukan model hingga bentuk bionomik. Asumsi dasar pemanenan yang dilakukan pada zona taman nasional laut, maka pemanenan dilakukan hanya terbatas. Pada populasi *prey one* diwilayahnya lebih dapat dipanen lebih banyak ketimbang pada population *prey two*. Secara ekologis population *prey one* hidup pada wilayah *caryying capacity* yang lebih besar. Pada sistem model predator-prey model predator-prey memungkinkan untuk memanen populasi yang hidup diwilayah yang lebih luas. Semakin luas daerah zona taman nasional laut maka semakin diperbolehkan untuk melakukan usaha eksploitasi semakin besar, dengan catatan tetap dibatasi.

Abstract

In the study of fisheries or marine products in developing countries, the problem of managing marine resources is often faced. Excessive exploitation due to weak legal and supervisory sectors is the most frequently used factor. This research involves a predator-prey mathematical model and provides an intervention variable exploitation, namely harvest. Harvest is carried out on two types of species that inhabit two protected areas of the marine national park zone. One of the objectives of the exploitation variable is to provide benefits for harvesters, such as fishermen. Boundary areas in the marine national park zone and points of equilibrium are assigned to research wetting. Stability analysis using the Routh-Hurwitz criteria indicates the survival of the population. The predator-prey model formed resulted in seven non-negative equilibria, but only one equilibrium point met the research assumptions. Numerical simulations are also provided in trajectories from the initial model formation to the bionomic shape. The basic assumption is that harvesting is carried out in the marine national park zone harvesting is carried out only in a limited way. In the prey one population, more can be harvested in the region than the prey two population. Ecologically, the population of prey one lives in a larger carrying capacity area. In the predator-prey model system, the predator-prey model makes it possible to harvest *populations that live in a wider area. The wider the area of the marine national park zone, the more it is permitted to carry out exploitation efforts, provided that it is still limited.*

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INTRODUCTION

Marine national park management zoning is an area that has limitations for exploitation activities. This zoning management is the first protective fence for the sustainability of ecosystem species. In the seas of tropical countries such as Indonesia, it has great potential for species survival. The marine national park zone managed by Indonesia has many potential species ecosystems (Sambali et al., 2015). Protection of ecosystems against harvesting exploitation is very strict and law enforcement is very good overall to protect marine national park zones. Each marine national park zone holds a lot of high marine resources (Ling & Johnson, 2012). The marine national park management zone is defined as an area protected by law or community custom (Micheli et al., 2004). The protection provided can be in the form of a ban on the exploitation of marine resources such as fish species, coral reefs, and seaweed. The no-exploitation approach is a popular policy in managing marine products (Pratama, 2022). The core function of marine national park zone protection is as a tool for environmental conservation and management. Policies issued by the government also make it easier to monitor the territorial sea zone. One of the effectiveness of protecting marine national park zones is through strict law enforcement and severe sanctions.

Marine park zones are considered an effective ecosystem-based conservation and management method worldwide. Many marine ecosystems have become extinct as a result of massive human exploitation. The process of extinction of marine species is generally divided into two statuses, namely critically endangered and endangered. Marine species with critically endangered categories in the marine national park zone include Chilatherina Sentaniensis, Encheloclarias Kelioides, Pandaka Pygmaea, and Marosatherina.

Ladigesi (Tobing et al., 2013)(Powell et al., 2015)(Fajarningsih et al., 2006)(Nasyrah et al., 2021)(Iskandar et al., 2021). The endangered species category in the marine national park zone includes Pterapogon Kauderni, Carcharhinus Borneensis, Scleropages Formosus, and Urogymnus Polylepis (Yazici & Bal, 2022)(C. Umar & Sulaiman, 2013)(Jimmi et al., 2011). Many species that are categorized as critically endangered and endangered are endemic species that live in the marine national park zone. Most of the species exploited are consumption species and ornamental species. The population numbers are decreasing every year, there is an urgent need for effective ecosystem management interventions (Kaligis et al., 2018). The protection of marine national park zone protects all ecosystem components in the area. Marine national park protection zones provide local-scale ecosystem-level protection (A. Umar & Musa, 2018). This protection is the strongest protection system, to protect the sustainability of the ecosystem of the national park area from excessive forms of human exploitation. Many previous studies stated that if for conditions of over exploitation, that is more than 35% of the carrying capacity and 20% exploitation in a protected zone, 30% of the strength of the protection effort. The yield unit time for the exploitation carried out can last from 20 years to 100 years.

The impact of exploitation activities on marine ecosystems is very large for humans and the species themselves. Over-exploitation can be carried out clandestinely in the marine zone of the national park. Application of the protected zone concept in marine national parks can protect and increase the number of species growing (Das et al., 2013). An increase in the number of species may occur in the area of the marine national park zone and areas outside the adjacent protected marine zone (Yokoo et al., 2012). Adjacent areas can be a means of species migration rates in the marine national park zone. Migration factors in the scope of the ecosystem, provide opportunities for exploited species to recover and increase the number of species.

The form of exploitation in this research is the continuous exploitation of species that are considered economically beneficial for humans. Economic benefits can include food and profits. Exploitation for the purpose of food needs tends to harvest constantly in small quantities. In exploitation with the aim of profit, it tends to force the maximum extraction of marine resources and reduce expenditure costs (Zhang et al., 2019). In larger cases, the form of exploitation is the harvesting of marine resources on an industrial scale or for industrial purposes. Therefore, the form of harvest species is more interesting to study in a mathematical model of the population in the marine national park zone area.

The predator-prey population model has become the subject of research in many relevant studies. Optimum harvest profit analysis is given to the population predator-prey model with stage-structure Monod-Haldane response function. Nonlinear harvesting of prey species has also been widely attributed to the predator-prey model (Kar & Chakraborty, 2010). The research results reveal that there is a reasonable upper bound on the level of the most profitable harvest over a long period of time. The population model is algebraically different from the population model system with harvesting efforts carried out on the prey population (Khan et al., 2021). Equilibrium point analysis using differential algebra is explored to see local stability. The dynamics of predator-prey interaction in fish with the intervention of bird predators have also occurred. The harvesting policy for both species uses the Pontryagin principle, and the trajectory analysis is validated through a numerical simulation (Pal & Mahapatra, 2014). The parameters taken vary in the realistic parameters to be considered which complement the concept of dimensions in the mathematical model. The various parameters tested show the results of the population growth trajectories. Confirmation of research results regarding dynamic reactions in marine resources, consisting of two important zones (Powell et al., 2015). The first zone is the free fish zone, which is located far from the shoreline or in the middle of the high seas.

The second zone is the protection zone, in this zone marine resources are not harvested indiscriminately. The harvest that is carried out can have a tendency to be consumed by local people. Catches in protected zones tend to be limited to certain species and the amount harvested is also limited. The type of population that is protected in a protected zone, functions as a source of storage to continue to revive the species that are harvested continuously (Chakraborty et al., 2011) (Kar & Chakraborty, 2010). Interactions in this kind of ecosystem are often found in many types of marine population ecosystems. A predator-prey model with a stage-structure that combines cannibalism in the nature of twin predators has also been discussed. The results of this study indicate that local equilibrium and stability are positive with the condition that there is a high "harvest" interaction. In this case, predators are harvested by exploitation and cannibalism among themselves.

Recent problems regarding mathematical models involving marine national park zones have been carried out with various analyzes (Li et al., 2017). A mathematical model that proposes a strategy for controlling marine national park zones from over-exploitation of prey species and reducing lost growth in each species (Belkhodja et al., 2018). In predator-prey ecological interactions, it always provides a fluctuating growth movement. Significant decline in populations in marine national park zones can lead to trade-offs between populations of main conservation species and management of certain species. Trophic interactions describe interaction pairs between species. The success of increasing the number of populations of one species can result in a decrease or even loss of valuable species that inhabit the marine national park zone.

METHOD

The research that was carried out was a type of literature study that involved a predator-prey mathematical model. The mathematical model is built from realistic assumptions to be considered based on the interactions in the ecosystem. Many research developments adopt the form of mathematical models built (Shang et al., 2021). After the mathematical model is built according to the basic concept assumptions, then a complete bioeconomic model is formulated. The mathematical model is extended to the problem of optimal control that takes into account the biological viability and economic objectives of managing ecological systems in marine national park zones. The classic predator-prey model, namely the Lotka–Volterra model (Islamiyati, 2019), is given to two prey populations. The shape of the Lotka–Volterra model as the basis for the population growth model is as follows,

$$\frac{dx}{dt} = rx - \alpha xy,$$
$$\frac{dy}{dt} = \beta xy - \delta y, \tag{1}$$

where, $x(t)$ is the population prey whose growth refers to the size of the population prey at time t. $y(t)$ is the size of the predator population at time t. Parameters r and δ are representative of the intrinsic growth rate of the prey and the mortality rate of predators. The rate of decline in the prey population as a result of predation is αxy. The increase in population as a result of predation is βxy. A measure of the biological efficiency of predation is given by the ratio $\frac{\beta}{\alpha}$.

The harvest rate follows the mathematical concept of catch versus effort,

$$h(x, E) = qEx. \tag{2}$$

In the proposed study, a mathematical model for harvest behavior gives the influence of marine national park zones, so that:

$$h(x, E, w) = wqEx, \quad 0 < w < 1. \tag{3}$$

wx is the number of species stocks available for harvesting in an ecosystem, while the shape $(1-w)$ is a mathematical model for the number of population stocks that are in adjacent protected zones. This research assumes that two prey populations are interdependent on each other in the life of the ecosystem. The modified mathematical model is as follows:

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K_1}\right) - wqEx,$$
$$\frac{dy}{dt} = ry\left(1 - \frac{y}{K_2}\right) - wqEy, \tag{4}$$
$$\frac{dE}{dt} = wqE(x-a) + wqE(y-b),$$

where, $x(t)$ represents the size of the population of prey one and $y(t)$ represents the size of the population of prey two. $E(t)$ is a harvesting effort made from the exploitation process. Population size $E(t)$ can also be thought of as the size of the predator-harvesting population. The growth rate of the prey population adopts the intrinsic growth model represented by the parameter is r. The catch coefficient is assumed to be the same for both prey populations, which is represented by the parameter q. K_1 and K_2 is the different carrying capacity of the population prey. One of the important basic assumptions in the developed mathematical model is that the spatial distribution of the population is uniform in all fishing grounds. In addition, it is also assumed that there is a population of prey that is vulnerable to harvesting. The shape a and b respectively is a critical level of population size economically, the condition of $(0 < a < K_1)$ and $(0 < b < K_2)$ if harvesting is done on prey species becomes unprofitable. The concept of this assumption is equivalent when the total population falls below that level, harvesters tend to leave the catchment zone.

The population in the marine national park zone is given in the equation $(1-w)$. Activities limit the catch of prey species based on season or location to control catch, bycatch, and fishing mortality. The harvest function is mathematically a partial harvest function $wqEx$ and $wqEy$ is an intervention in the exploitation model (4). Model (4) can become a Schaefer model if conditions are reduced at $w = 1$. Justify the assumption that population growth of prey species grows logistically when there is no fishing. While exploitation activities that cause a decrease in the number of populations are given Ex and Ey for each prey species. The next basic assumption is an attempt to restore the ideal condition of the number of population which decreases exponentially when $x<a$ and $y<b$. Mathematically it is clear that, if the condition is $x=a$ atau $y=b$, then the exploitation effort is zero. The fishing variable represented by $E(t)$ is a dynamic variable which is an important variable in model (4). This concept is different from the Gordon-Schaefer bioeconomic model, which assumes that effort is always constant. All the variables and parameters that make up the model (4) are shown in the following table,

Table 1. Variables and Parameter Description

Symbol	Description	Unit
x	Prey one population (time dependent),	$[N]$
y	Prey two populations (time dependent),	$[N]$
E	Harevsting effort (time dependent),	-
r	Prey's one intrinsic growth rate,	$[T]^{-1}$
w	Number of prey species stocks,	-
q	Catch coefficient,	$[T]^{-1}$
a	Critical level of population size economically,	$[N]$
b	Critical level of size of the population economically,	$[N]$
K_1	Carrying capacity of prey one,	-
K_2	Carrying capacity of prey two.	-

RESULTS AND DISCUSSION

1. Equilibrium Analysis

The model (4) is analyzed using the linearization of differential equations, namely $\frac{dx}{dt}=0$, $\frac{dy}{dt}=0$ and $\frac{dE}{dt}=0$. Variable species prey and each catch effort will show a solution. The solution taken is non-negative because it is realistic to consider the survival of the species. Testing the stability of the model solution (4) is to use the Jacobian matrix. Characteristic equations and eigenvalues will also be shown in this section. The linearization equation of model (4) is as follows,

$$
\begin{aligned}
&rx\left(1-\frac{x}{K_1}\right)-wqEx=0,\\
&ry\left(1-\frac{y}{K_2}\right)-wqEy=0,\\
&wqE(x-a)+wqE(y-b)=0,
\end{aligned}
\tag{5}
$$

The overall solution of model (4) includes non-negative solutions including;
$P_0(0,0,0)$, $P_1(x_1,0,0)$, $P_2(0,y_2,0)$, $P_3(x_3,y_3,0)$, $P_4(0,y_4,E_4)$, $P_5(x_5,0,E_5)$ and $P_6(x_6,y_6,E_6)$.
where,

$$x_1=x_3=K_1,\quad y_2=y_3=K_2,\quad y_4=x_5=a+b,\quad E_4=-\frac{r(a+b-K_2)}{wqK_2},\quad E_5=-\frac{r(a+b-K_1)}{wqK_1},$$

$$x_6=\frac{K_1(a+b)}{K_1+K_2},\ y_6=\frac{K_2(a+b)}{K_1+K_2},\ E_6=-\frac{r(a+b-K_1-K_2)}{wq(K_1+K_2)}.$$

The equilibrium points that appear as many as seven non-negative equilibriums, but what is realistic for stability analysis to do are equilibrium points $P_6(x_6,y_6,E_6)$. The remaining equilibrium point is the equilibrium point which has a zero value for each population, of course, this cannot be analyzed in detail, therefore the equilibrium P_6 point is the most realistic equilibrium point to consider. The survival of the species and positive harvest values will help study the research results. In ecosystem life, behavior like this is commonplace, namely maintaining the sustainability of the population of prey species and obtaining benefits from the harvest business carried out. The Jacobian matrix corresponding to model (4) is as follows,

$$J(P_i)=\begin{bmatrix} j_{11} & 0 & j_{13} \\ 0 & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33} \end{bmatrix}, \tag{6}$$

where,

$j_{11}=r\left(1-\dfrac{x}{K_1}\right)-\dfrac{rx}{K_1}-wqE$, $j_{13}=-wqx$, $\quad j_{22}=r\left(1-\dfrac{y}{K_2}\right)-\dfrac{ry}{K_2}-wqE$, $\quad j_{23}=-wqy$,

$j_{31}=wqE$, $j_{32}=wqE$, $j_{33}=wq(x-a)+wq(x-b)$.

The equilibrium points $E_6(x_6,y_6,E_6)$ corresponding to the Jacobian matrix (3) are as follows,

$$J(P_6)=\begin{bmatrix} j_{11} & 0 & j_{13} \\ 0 & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33} \end{bmatrix}, \tag{7}$$

where,

$j_{11}=r\left(1-\dfrac{x_6}{K_1}\right)-\dfrac{rx_6}{K_1}-wqE$, $j_{13}=-wqx_6$, $\quad j_{22}=r\left(1-\dfrac{y_6}{K_2}\right)-\dfrac{ry_6}{K_2}-wqE$, $\quad j_{23}=-wqy_6$,

$j_{31}=wqE$, $j_{32}=wqE$, $j_{33}=wq(x_6-a)+wq(y_6-b)$.

The characteristic equations associated with the Jacobian matrix $J(P_6)$ are,

$$f(\gamma)=\gamma^3+N_0\gamma^2+N_1\gamma+N_2, \tag{8}$$

where,

$$N_1=-\frac{-2arK_1-2arK_2-2brK_1-2brK_2}{(K_1+K_2)^2},$$

$$N_2=\frac{\left(a^2qrwK_1+a^2qrwK_2+2abqrwK_1+2abqrwK_2-aqrwK_1{}^2\right)}{(K_1+K_2)^2}+$$
$$\frac{\left(-2aqrwK_1K_2-aqrwK_2{}^2\right)}{(K_1+K_2)^2}+$$
$$\frac{\left(b^2qrwK_1+b^2qrwK_2-bqrwK_1{}^2-2bqrwK_1K_2-bqrwK_2{}^2-a^2r^2-2abr^2-b^2r^2\right)}{(K_1+K_2)^2},$$

$$N_3=\frac{\left(a^3qr^2w+3a^2bqr^2w-a^2qr^2wK_1-a^2qr^2wK_2+3ab^2qr^2w-2abqr^2wK_1\right)}{(K_1+K_2)^2}+$$
$$\frac{1}{(K_1+K_2)^2}\left(-2abqr^2wK_2+b^3qr^2w-b^2qr^2wK_1-b^2qr^2wK_2\right).$$

The Routh-Hurwitz criterion, the eigenvalue of the characteristic polynomial equation eq. (8) has negative real parts if $N_0>0$, $N_1>0$ and $N_2>0$,

$\Delta_1 = |N_0| > 0$,

$\Delta_2 = \begin{vmatrix} N_0 & N_3 \\ 1 & N_2 \end{vmatrix} > 0$,

Where $N_0 > 0$, $N_1 > 0$ and $N_0 N_1 - N_2 > 0$. Therefore, the eigenvalues γ_1, γ_2 and γ_3 belong to the set,

$$\{(\gamma_1, \gamma_2, \gamma_3) \in R_3 : \gamma_1 < \gamma_2 < \gamma_3 < 0\}, \quad (9)$$

2. Bionomic Equilibrium

The integration of biological variables and economic variables in a stable predator-prey model is a basic concept of bionomics. The total sustainable income from exploitation is equal to the total business costs incurred, so simply a bionomic process has occurred. Sustainable net income is obtained from the total price per tonne of catch and the cost per trip of exploitation business. Bionomic balance is obtained by completing a mathematical model with the condition that the population species do not experience growth or the growth rate is zero. The profit function obtained from the harvesting business is formed from the control function and the total cost function. The variable p is the price per unit ton of fish landed or fish harvested and c is the total cost per fishing trip, the net sustainable profit is given by the following form,

$$\pi(x, E, u) = (pwqx - c), \quad (10)$$

The assumes that the price per unit ton of fish tends to be stable so that the p value will tend to be constant. The assumption is reversed for the cost per fishing trip, due to faster fluctuations and inflation, such as increases in fuel, food ingredients, and others. Harvesting predator and prey fish species, the total cost can be $c = c_1 + c_2$.

The bionomic mathematical model is as follows;

$$\frac{dx}{dt} = \frac{dE}{dt} = \pi = 0, \quad (11)$$

The form of a homogeneous equation solving solution from a linear equation form is as follows:

$$rx\left(1 - \frac{x}{K_1}\right) - wqEx = 0,$$

$$ry\left(1 - \frac{y}{K_2}\right) - wqEy = 0, \quad (12)$$

$$wqE(x - a) + wqE(y - b) = 0,$$

$$(pwqx - c)E = 0.$$

where, $pwqx < c$ if the cost of the fishing effort exceeds the gross income, then the result can be obtained that the net income for the harvesting business is negative. The bionomic balance assumes that $pwqx > c$.

The bionomic balance point is symbolized by $P_\infty(x_\infty, y_\infty, E_\infty, w_\infty)$, which involves all exploitation efforts, namely harvesting predators and prey, where

$$x_\infty = \frac{K_1(a + b)}{K_1 + K_2},$$

$$y_\infty = \frac{K_2(a + b)}{K_1 + K_2},$$

$$E_\infty = \frac{rp(a + b - K_1 + K_2)(a + b)}{(c_1 + c_2)(K_1 + K_2)},$$

$$w_\infty = \frac{(c_1 + c_2)}{pq(a+b)}.$$

The concept of marine fisheries management needs to consider the economic interests of the harvesting business with the aim of sustaining ecological resources. For example, if the stock of marine species is too large, the profits from harvesting will be reduced. Meanwhile, if the population reserves are very small or close to depleted stocks, it will certainly not achieve the goal of sustainable resources. The bionomic concept requires careful calculations to prepare a population species reserve. In cases that occur in general, for example, if the condition $w < w_\infty$ of economic interest becomes negative, this indicates that fishing is no longer feasible and harvesting is no longer carried out in that area. Contrary, if there $w > w_\infty$ is positive economic interest, this indicates that the fishing effort is feasible so that conditions of over-exploitation can occur, thus attracting more fishing actors into the area.

3. Numerical Simulation

Growth fluctuation system of population species for models of predator-prey can be analyzed by using trajectories. Numerical simulation provides a simple description of the movement of population growth over time. The optimum harvesting theory is carried out by providing iterations that reach convergence. The assumption of parameter values used in the numerical simulation is based on several references and assumptions. The analysis in this section also shows the equilibrium point which includes model (4). Parameters in models (4) are given as follows $r = 0.1$, $K_1 = 199$, $K_2 = 100$, $u = 0.7$, $a = 0.1$, $b = 0.8$, $q = 0.0000008$, and $w = 0.000003$. There are seven positive equilibrium points corresponding to model (4), each of which is as follows $P_0(0,0,0)$, $P_1(199,0,0)$, $P_2(0,100,0)$, $P_3(199,100,0)$, $P_4(0,0.9,1.12\times 10^7)$, $P_5(0.9,0,1.12\times 10^7)$ and $P_6(0.5989966555, 0.3010033445, 1.123591867\times 10^7)$.

All the equilibria that appear are equilibrium points that meet the positive point qualifications. Of the seven equilibrium points that support model (4), only one equilibrium point is realistic to consider, namely $P_6(0.5989966555, 0.3010033445, 1.123591867\times 10^7)$. Equilibrium is P_6 calculated for the stability of model (4) with Routh-Hurwitz criteria. The characteristic equation that appears is,

$$\lambda^3 + 0.00854849\lambda^2 + 0.0000184297307\lambda + 6.862017196\times 10^{-6} = 0, \qquad (12)$$

with the eigenvalues that appear are $\lambda_1 = -0.00423635039435312$, $\lambda_2 = -0.0000378966056468789$, and $\lambda_3 = -0.004274247$. The asymptotic local stability of model (4) is achieved under conditions of eigenvalues $\lambda_1 < 0$, $\lambda_2 < 0$, and $\lambda_3 < 0$.

The harvesting model that has been given in model (11) to obtain maximum profit and at the same time carry out the sustainability of the species population. The bionomic equilibrium model (11) given is $P_\infty(x_\infty, y_\infty, E_\infty, w_\infty) = (0.5989966555, 0.3010033445, 0.01019322542, 7.716049383\times 10^8)$. In the theory of fishing effort and economic feasibility, the proportion of the available population must be greater than 60%. At the level of marine resources stored at most 40% assuming that other parameters remain unchanged.

In a further simulation, simulation trajectories are given which describe the shape of the population growth movement. Simulations trajectories take the initial equilibrium values, $x(0) = 0.5$, $y(0) = 0.3$, and $E(0) = 10$. The values of this approach are taken close to the serious value of the equilibrium point in the model (4). Model (4) meets the form of asymptotic locally stable asymptotic population predator-prey model, such as the following trajectories;

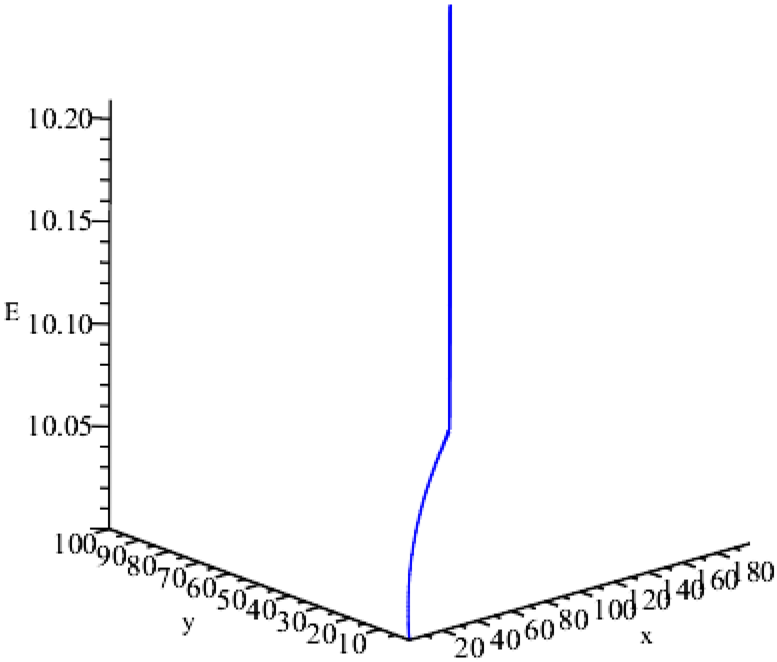

Figure 1. 3D of the system (4) stable local points.

In the case of fig.1 it can be seen that the equilibrium fluctuates linearly at points along the equilibrium values. The variable that intervenes in a model (4), namely the harvest variable, has a slow growth effect. Things like this in the ecosystem can occur because harvesting is done on the parent species, thereby reducing the birth of new species. Effect harvest on each population is different.prey x and y move strongly, but the growing movement is dominated by the population species y. Meanwhile, the population species x is under pressure so its growth tends to be slow compared to the population species y. The trajectories of each population grow towards the equilibrium model point (4), which trajectories can be seen as follows;

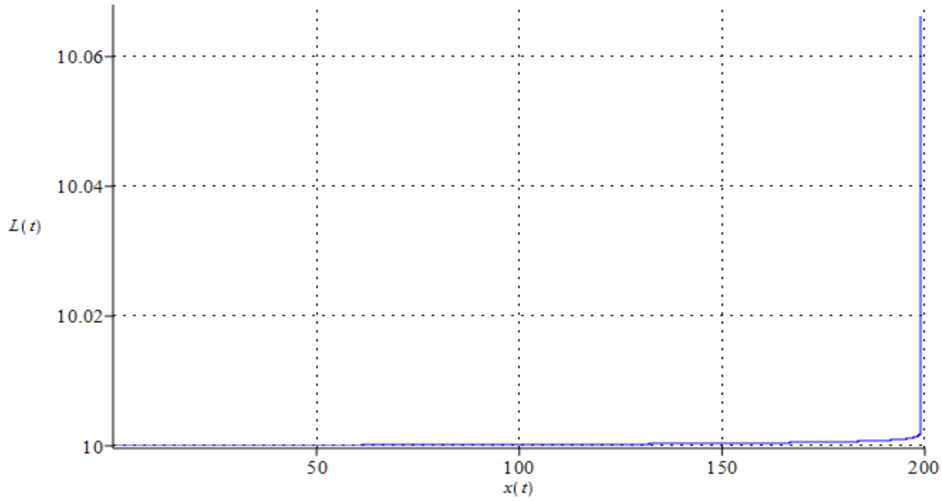

Figure 2. Trajectories Population Prey One (x)

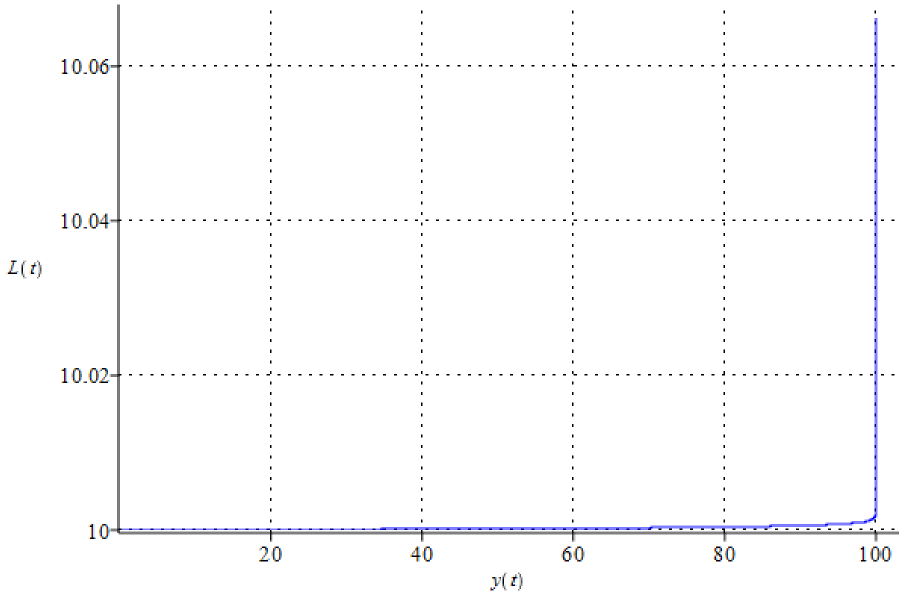

Figure 3. Trajectories Population Prey two (y)

Trajectories in Figure 2 and Figure 3 show the movement of population species growth towards harvesting efforts. The harvesting effort given in model (4) can give us an idea that the movement of the harvesting business can provide maximum profit. Returning to the basic concept of harvesting in the marine national park zone, harvesting is limited. The number of populations for each species has a maximum limit for the number of populations. This maximum population limit will protect the existing system in the marine national park zone. In the prey one population, more can be harvested in the region than in the prey two population. Ecologically, the population of prey one lives in a larger carrying capacity area. In the predator-prey model system (4) it is possible to harvest populations that live in a wider area. The wider the area of the marine national park zone, the more it is permitted to carry out exploitation efforts, provided that it is still limited. Likewise for the population that lives in the marine national park zone with a small size, the harvesting that is carried out is also limited or not maximum in profit. In essence, if the cost of the harvesting business exceeds the gross profit, then the exploitation effort should not be carried out, because it does not provide benefits for the exploiters.

CONCLUSION

This study considers harvest in a predator-prey model with a reserve population in the marine national park zone. The model is involved in stability analysis by first finding equilibrium. The characteristic equation for determining local stability is also shown. The model developed is model (4) by adopting a predator-prey mathematical modeling structure. All equilibrium points that appear in a model (4) include non-negative solutions including; $P_0(0,0,0)$, $P_1(x_1,0,0)$, $P_2(0,y_2,0)$, $P_3(x_3,y_3,0)$, $P_4(0,y_4,E_4)$, $P_5(x_5,0,E_5)$ and $P_6(x_6,y_6,E_6)$. Seven realistic equilibrium points for stability analysis are equilibrium points $P_6(x_6,y_6,E_6)$. The analysis is carried out by involving exploitation effort points that are integrated with the model. Numerical simulation analysis is also given to see the trajectories of population growth movement. In the numerical simulation, an initial value is given which represents the basic assumptions of the model. A stable non-negative equilibrium is obtained $P_6(0.5989966555, 0.3010033445, 1.123591867\times 10^7)$. The equilibrium point P_6 is tested using Routh-Hurwitz criteria to see sustainable stability. Numerical simulations are also provided in the bionomic model analysis (4). Maximum profit and at the same time the survival of the species population. The bionomic equilibrium model (11) given is $P_\infty(x_\infty,y_\infty,E_\infty,w_\infty)=(0.5989966555, 0.3010033445, 0.01019322542, 7.716049383\times 10^8)$. In the theory of fishing effort and economic feasibility, the proportion of the available population must be greater than 60%. At the level of marine resources stored at most 40% assuming that other parameters remain unchanged. In future research, the analysis will be improved in cases that have limited harvest and restricted migration areas. Assumptions can be built to see the reality of ecosystem bioecology.

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